FLAT-PROBE METHOD FOR DETERMINATION OF THE THERMOPHYSICAL PROPERTIES OF MASSIVE MATERIALS

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A method of nondestructive measurement of the thermophysical properties of massive materials is proposed in which the heat leakage of the heater to the surrounding medium is accounted for and the distorting influence of its heat capacity on the temperature field of the sample is eliminated.

Recently methods of nondestructive testing of the thermal conductivity and thermal diffusivity of materials have become widely applied in thermophysical measurements. Their specific feature lies in the fact that there is no need to incorporate a probe into the tested material, and the sample may be of arbitrary form but must have one of its sides smooth. Determination of thermophysical properties is reduced to supply of a constant heat flux via a spot of the contact in the form of a circle of definite diameter to the surface of a body semiinfinite in thermal aspect and measurement of the temperature variation in time.

Many variants of the above method have been proposed. The majority of them are based on the principle of absence of heat leakage into the surrounding medium [1-8], whose implementation requires utilization of additional technical means for automatic temperature control of the medium. In [9] a method is proposed excluding heat leakage into the surrounding medium since the heat source is positioned at the joint of two similar semiinfinite bodies. However, the identity of the investigated samples substantially restricts the sphere of applicability of the method. For instance, it cannot be used for field measurements of heat coefficients of massifs of soils and rocks or other materials. In this respect, the method proposed by G. M. Serykh and B. A. Gergesov [10] is of interest. It employs a system of two composite bodies. The thermophysical properties of a material are measured in the initial heating region (Fo < 0.1), which ensures a short time of measurements, which is undoubtedly a merit of the method. At the same time in the initial heating regime the distorting influence of the heat capacity of the heater on the temperature field of the material may manifest itself, which causes additional errors in the experiment. Manufacture of a low-response heater presents technological difficulties. In [11] the heat capacity of the heater is accounted for on the surface of one half-space.

The distortions of the temperature field of a sample due to the heat capacity of the heater may be avoided by choosing a final stage of heating where the regime of the linear dependence of temperature on the parameter $1/\sqrt{\tau}$ reaches the steady state in the temperature field [7, 8]. In the present article this principle is extended to a system composed of two semiinfinite bodies, which allows a decrease of the experimental error at the expense of elimination of the distorting influence of the heat capacity of the heater on the temperature field of the sample and an account of heat leakage of the heater into the surrounding medium via the second body with known heat coefficients. As the latter a heat-insulation layer may be chosen, owing to which the experimental setup becomes self-contained and may be used for field measurements of the thermophysical properties of rocks or other materials.

The essence of the proposed material is as follows. A flat heat source having constant heating power and the configuration of a circle with radius R acts between two semiinfinite bodies with different thermophysical properties having an ideal thermal contact. Thus, a heat flux with the constant power $q = \text{const}$ is produced on the part of the contact with radius R, while on the remaining part of the contact the flux is absent.

In the plane $z = 0$ heat transfer between bodies is negligible and the heat fluxes q_1 and q_2 are completely directed into each body from the heat source but depend on the coordinate r. The sum of these fluxes gives the heat flux produced by the heat source q, i.e., $q_1 + q_2 = q$.

At the initial moment $\tau = 0$ the constant temperature T_0 is maintained throughout the whole volume of the bodies.

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Then we may write the following system of differential equations of heat conduction:

$$\frac{\partial T_1}{\partial \tau} = a_1\left(\frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r}\frac{\partial T_1}{\partial r} + \frac{\partial^2 T_1}{\partial z^2}\right),$$
$$\frac{\partial T_2}{\partial \tau} = a_2\left(\frac{\partial^2 T_2}{\partial r^2} + \frac{1}{r}\frac{\partial T_2}{\partial r} + \frac{\partial^2 T_2}{\partial z^2}\right), \tag{1}$$

where a_1 and a_2 are the thermal diffusivities of the first and second bodies, with the following boundary conditions:

at $0 \le r \le R$

$$\frac{\partial T_1(r,\ 0,\ \tau)}{\partial z} = -\frac{q_1}{\lambda_1};$$
$$\frac{\partial T_2(r,\ 0,\ \tau)}{\partial z} = -\frac{q_2}{\lambda_2};$$
$$q_1 + q_2 = q,$$

at $r \ge R$

$$\frac{\partial T_1(r,\ 0,\ \tau)}{\partial r} = \frac{\partial T_2(r,\ 0,\ \tau)}{\partial r} = 0;$$
$$T_1(r,\ z,\ 0) = T_2(r,\ z,\ 0) = T_0;$$
$$T_1(r,\ \infty,\ \tau) = T_2(z,\ \infty,\ \tau) = T_0; \tag{2}$$
$$\frac{\partial T_1(r,\ \infty,\ \tau)}{\partial r} = \frac{\partial T_2(r,\ \infty,\ \tau)}{\partial z} = 0;$$
$$T_1(r,\ 0,\ \tau) = T_2(r,\ 0,\ \tau),$$

where λ_1 and λ_2 are the thermal conductivities of the first and second bodies, respectively.

The solution of this problem for the excess mean-integrated temperature ϑ_τ of the heating zone contact in the form of a circle after some elapse of time is as follows:

$$\bar{\vartheta}_\tau = \frac{8}{3\pi}\frac{qR}{\lambda_1 + \lambda_2} - \frac{qR^2}{2\sqrt{\pi}(\lambda_1\sqrt{a_1} + \lambda_2\sqrt{a_2})}\frac{1}{\sqrt{\tau}}, \tag{3}$$

where $\vartheta_\tau = \bar{T}_\tau - T_0$; $\bar{T}_\tau$ is the mean-integrated temperature of the contact spot of heating up.

Relation (3) allows determination of the thermal conductivity and thermal diffusivity by the excess mean-integrated temperature of the heating contact spot $\vartheta_\tau(1/\angle\tau)$, then the latter will approach a linear dependence in the course of time (Fig. 1): here intersection of the straight line with the ordinate axis $(1/\angle\tau = 0)$ gives the value of the temperature difference under steady-state heating conditions $(\tau \to \infty)$, which may be used to calculate the thermal conductivity of the sample

$$\lambda_1 = \frac{8qR}{3\pi\,\bar{\vartheta}_{\text{ст}}} - \lambda_2, \tag{4}$$

where $\varepsilon_{st} = \bar{T}_{st} - T_0$ is the steady-state difference between the mean-integrated temperature of the heat source and the initial temperature of the sample.

The thermal diffusivity may be determined either by the intersection of the straight dependence $\vartheta_\tau(1\sqrt{\tau})$ with the abscissa $1\sqrt{\tau_1}$ or by its tangent φ

$$a_1 = \left[\frac{3\sqrt{\pi}R(1 + \lambda_2/\lambda_1)}{16}\frac{1}{\sqrt{\tau_1}} - \frac{\lambda_2\sqrt{a_2}}{\lambda_1}\right]^2, \tag{5}$$

$$a_1 = \left[\frac{qR^2}{2\sqrt{\pi}\lambda_1\,\mathrm{tg}\,\varphi} - \frac{\lambda_2\sqrt{a_2}}{\lambda_1}\right]^2. \tag{6}$$

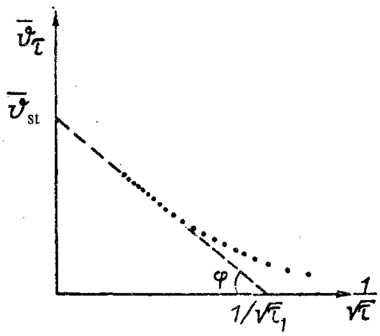

Fig. 1

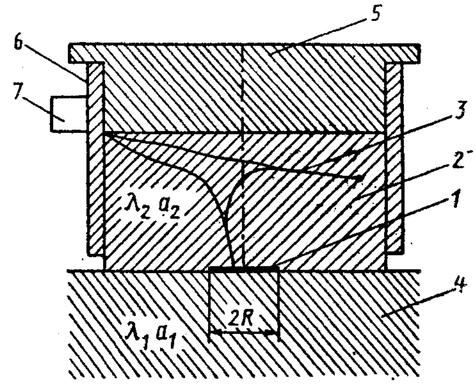

Fig. 2

Fig. 1. Plot of the excess mean-integrated temperature of the heater ϑ_τ (K) vs $1\sqrt{\tau}$ (sec$^{-1/2}$).

Fig. 2. Block diagram of the flat probe.

A block diagram of the probe is shown in Fig. 2. The probe consists of flat heating element 1, base-heat insulator 2, and copper–constantan differential thermocouple 3. The heating element is assembled on a copper foil in the form of a circle with a definite radius. To its inner surface a 0.1-mm diameter nichrome heating spiral, uniformly arranged over the whole plate surface, is glued. The "hot" junction of the thermocouple is soldered to the foil. Its "cold" junction is placed at a sufficient distance in heat-insulating porous rubber with $\lambda \approx 0.05$ W/(m·K) and $a_1 \approx 2.22 \cdot 10^{-7}$ m^2/sec. During the experiment the temperature of the "cold" junction must not change by more than 0.005 K. The copper body of the heater allows measurement of the practically averaged temperature of the contact heating zone by the "hot" junction. For the sake of convenience of utilization the heater is glued to the heat-insulation surface. Tight thermal contact of sample 4 with the heat insulation is achieved by greasing the contact surface with technical vaseline and its compression by cover 5. All the units are placed in body 6 with joint 7.

Now we estimate the distorting influence of the heater heat capacity on the temperature field of the system of bodies. For this, we start with the heat balance equation of the heater

$$q_h = \frac{cm_h}{S_h}\frac{dT_h}{d\tau} + q_1 + q_2, \tag{7}$$

where T_h, cm_h, S_h, and q_h are the temperature, heat capacity, surface area, and heat flux of the heater.

In calculations, use is made of the heat flux produced by the heater, which is taken as theoretical (q_{th}). It is necessary that the real heat flux to the system of bodies $q_{real} = q_1 + q_2$ be equal to q_{th} with the permissible error ϵ_c. Their inequality is conditioned by the heat capacity effect of the heater. Thus we have

$$\varepsilon_c = \left(1 - \frac{q_{real}}{q_{th}}\right) = \frac{cm_h}{q_{th}\, S_h}\frac{dT_h}{d\tau}. \tag{8}$$

The derivative $dT_h/d\tau$ is found from formula (3) at $\tau \geq \tau_{lin}$ (here τ_{lin} is the time of onset of the linear dependence of the temperature on $1\sqrt{\tau}$ with the permissible error ϵ_{lin}). Then taking account of the relations $S_h = \pi R^2$, $cm_h = C_{\nu h}V_h = C_{\nu h}\pi R^2 l$ (here R and l are the radius and the thickness of the heater) we may write

$$\varepsilon_c = A\frac{R^2 l}{\tau^{3/2}}, \tag{9}$$

where

$$A = \frac{c_{\gamma h}}{4\sqrt{\pi}\lambda_1 a_1^{1/2}\,[1 + (\lambda_2/\lambda_1)(a_2/a_1)^{1/2}]}$$

is a constant dependent on the thermophysical properties of the heater and the system of semiinfinite bodies.

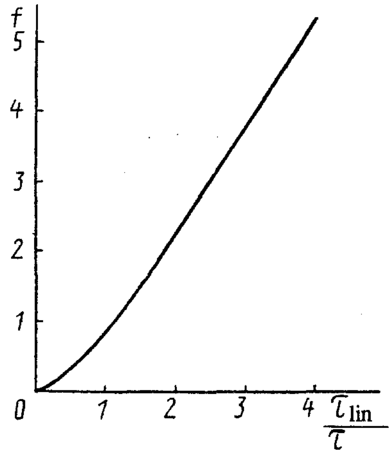

Fig. 3. Function f vs τ_{lin}/τ.

TABLE 1. Thermal Conductivity Determined by the Methods of Flat and Cylindrical Probes

Material	Volume weight of skeleton, kg/m^3	Weight moisture contents, %	Thermal conductivity, W/(m · K)	
			flat probe	cylindrical probe
Frozen sand	1670	20.2	2.96	3.09
Argillaceous marl	2160	10.2	2.14	2.13
Ice	922	–	2.24	2.30

Thus, the distorting effect of the heat capacity of the heater ϵ_c depends on the heater dimensions and time. As $l \to 0$, $\epsilon_c \to 0$. For the given dimensions of the heater the parameter ϵ_c rapidly decreases with increase of time since it depends on time to a greater degree than the temperature itself.

From (9) we may find the time of disappearance of the distorting effect of the heat capacity of the heater on temperature field of the sample with the permissible error ϵ_c:

$$\tau_c \geqslant \sqrt[3]{\left(\frac{AR^2 l}{\varepsilon_c}\right)^2}. \tag{10}$$

As a consequence, we consider the following cases.

1. $\lambda_2/\lambda_1 \to 0$, i.e., ideal heat insulation. Here we have

$$A_1 = \frac{c_V h}{4\sqrt{\pi}\lambda_1 a_1^{1/2}}. \tag{11}$$

2. $\lambda_1 = \lambda_2$ and $a_1 = a_2$, i.e., pair samples. Then

$$A_2 = \frac{c_V h}{8\sqrt{\pi}\lambda_1 a_1^{1/2}} = \frac{A_1}{2}. \tag{12}$$

3. $\lambda_1 \neq \lambda_2$ and $a_1 \neq a_2$. The constant A_3 is determined by formula (9). In this case we obtain that $\tau_{c2} \leq \tau_{c3} \leq \tau_{c1}$.

Thus, τ_c undergoes a twofold change between the two extreme cases. In the case of ideal heat insulation of the heater we have the maximum time of cessation of the heat capacity effect of the heater on the temperature field of the system τ_{cmax} and in estimations it should be used as the worst experimental conditions. Depending on the heater heat capacity this time may be longer or shorter than that of the onset of the linear section τ_{lin} of the temperature dependence on the parameter $1\sqrt{\tau}$. The quantity τ_{lin} has not been evaluated above since the solution (3) is derived for the already attained linear dependence $T \to f(1\sqrt{\tau})$. Proceeding from the worst experimental conditions, let us consider the case of ideal heat insulation of the heater in the form of a circle with radius R on the surface of semiinfinite body 1. In this case the excess heater temperature at its center at $R/2(a_1\tau)^{1/2} < 1$ is described by the equation [7]

$$\vartheta_\tau = \frac{qR}{\lambda_1}\left\{1 - \frac{R}{2\sqrt{\pi a_1 \tau}}\left(1 - \frac{R^2}{24 a_1 \tau} + \frac{R^4}{480 a_1^2 \tau^2} - \ldots\right)\right\}. \tag{13}$$

In order to establish the linear dependence of ϑ_τ on the parameter $1\sqrt{\tau}$, it is necessary for the expression in round brackets in (13) not to exceed $1 - \epsilon_{lin}$.

We assume $\epsilon_{lin} \approx 0.04$. Then $R^2/a_1\tau \leq 1$. Hence the time of attaining the linear dependence is $\tau_{lin} \geq R^2/a_1$.

In subsequent calculations the derivative $dT_h/d\tau$ in (7) is found from (13):

$$\frac{dT_h}{d\tau} = \frac{q_{th}\, R^2}{4\sqrt{\pi a_1}\lambda_1 \tau^{3/2}}\left(1 - \frac{R^2}{8 a_1 \tau} + \frac{R^4}{96 a_1^2 \tau^2} - \ldots\right). \tag{14}$$

Taking into account $\tau_{lin} \geq R^2/a_1$, we finally arrive at

$$\varepsilon_c = \frac{Bl}{R} f\left(\frac{\tau_{lin}}{\tau}\right), \tag{15}$$

where $B = (1\sqrt{\pi})\cdot C_{\gamma h}/C_{\nu 1}$ is a new constant;

$$f\left(\frac{\tau_{lin}}{\tau}\right) = \left(\frac{\tau_{lin}}{\tau}\right)^{3/2}\left[1 - \frac{1}{8}\frac{\tau_{lin}}{\tau} + \frac{1}{96}\left(\frac{\tau_{lin}}{\tau}\right)^2 - \ldots\right].$$

Since formula (13) is valid at $R/2(a_1\tau)^{1/2} < 1$, the function $f(\tau_{lin}/\tau)$ is to be calculated by (15) within the limit $\tau_{lin}/\tau \leq 4$ (Fig. 3). As $l \to 0$ and $\tau \to \infty$, $\epsilon_c \to 0$. For the given heater dimensions ($l/R \neq 0$) the quantity ϵ_c is determined by the parameter τ_{lin}/τ. In the initial heating region ($\tau_{lin}/\tau > 1$) the distortion is at its maximum. For instance, at Fo ≤ 0.1, which is recommended in [10] and corresponds to $\tau_{lin}/\tau \geq 10$, the value of f is less than 15. In the linear region of temperature variation with the parameter $1\sqrt{\tau}$ the value of f does not exceed 0.9.

Now we examine ϵ_c with reference to concrete examples.

Let the heater body be manufactured from a 1-mm-thick copper sheet and the heater diameter be 30 mm. Then the volume heat capacity of the heater is equal to 3738 kJ/($m^3\cdot$K). As the tested material, we consider a soil mass. Its volume heat capacity in dry and moist, thawed, frozen states varies from 820 to 4200 kJ/($m^3\cdot$K). Rocks, ice, concrete and, practically, all the materials fall within these limits. Here we have $C_{\nu h}/C_{\nu 1} = 1$-4. Then B = 0.14-0.56 and $l/R = 0.067$. In this case we have $\epsilon_c = (0.009\text{-}0.037)f(\tau_{lin}/\tau)$. Of these bounds, we choose the worst one, i.e., $\epsilon_{cmax} = 0.037f(\tau_{lin}/\tau)$. For different τ_{lin}/τ (0.3; 0.5; 0.8; 1.0; 2.0; 3.0, and 4.0) ϵ_{cmax} varies as follows: 0.007; 0.013; 0.025; 0.035; 0.082; 0.137, and 0.200. Hence it is obvious that for the accepted heater dimensions in the linear region of the dependence $T \to f(1\sqrt{\tau})$ the distorting effect of the heater does not exceed 3.5%. For $\tau_{lin}/\tau \leq 10$ (Fo ≤ 0.1) the value of ϵ_c is less than 50%.

Thus, the choice of the steady-state regime of the linear temperature dependence on the parameter $1\sqrt{\tau}$ in measurements of thermophysical properties of materials by the proposed method allows one to substantially decrease the experimental errors associated with the distorting effect of the heat capacity of the heater on the temperature field of the tested sample.

The error in determination of thermophysical characteristics by the proposed method is evaluated by comparison with the results of measurements of these characteristics by the method of a cylindrical probe with a constant heating power (the probe-needle). The measurements performed in underground chamber at a constant natural temperature of 269.8 K for masses of frozen sand, ice and argillaceous marl. The results obtained are listed in Table 1.

As seen from the table, the thermal conductivity data obtained by the methods of flat and cylindrical probes agree within ±5-6%. For thermal diffusivity, deviations of the measured data from tabular data (for ice $a = 1.16\cdot10^{-6}$ m²/sec) attain 13-15%.

NOTATION

r, z, current coordinates; τ, time; T_τ, temperature of the semiinfinite body at any moment of time; T_0, initial temperature of the body; $\vartheta_\tau = T_\tau - T_0$, excess temperature at the heater center; $\vartheta_\tau = \bar{T}_\tau - T_0$, mean-integrated temperature of the heater; λ, thermal conductivity; a, thermal diffusivity; C_γ, volume heat capacity; c, specific heat capacity; q, heat flux; S_h, heater area; R, its radius; ϵ, permissible error; φ, slope of the linear temperature dependence on the parameter $1\sqrt{\tau}$. The indices 1, 2, h, real, th, c, lin denote the tested sample, reference body, heater, real and theoretical heat flux, heat capacity of the heater and linear law of temperature variation with the parameter $1\sqrt{\tau}$.

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